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Specifying algebraic data types by domain equations

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ABSTRACT - The paper provides the theoretical foundation for a new algebraic specification method, using parameterized specifications and algebraic domain equations, an algebraic analogon to the domain equations used in Scott's theory of data types. The main result is that algebraic domain equations always have an initial solution. Also, a parametric version of algebraic domain equations is investigated. In either case, there is a simple syntactic solution method.

1. INTRODUCTION

Scott's order theoretic approach to data types (SC 71 ff) offers a characteristic specification method: there are given basic types like BOOL, INT, etc., and there is a fixed set of type constructors like sum, product, function space, and powerdomain. Data types are then defined by so-called domain equations that may be recursive, e.g. (cf. SC 72a, SP 77) $N \cong N+1$ (natural numbers), $L \cong (L \times L)+D$ (list structures over D), $F \cong [F \longrightarrow F]$ (a pure λ-calculus model), $S \cong S \times D+1$ (stacks over D), etc.

In contrast, the algebraic approach to data types has hardly more to offer than explicit specifications: one has to give all sorts, all operations, and all axioms that describe the behaviour of the operations (cf. ADJ 78). Some caution is required when comparing the order-theoretic and algebraic approaches to data types, since they do not model the same aspects and serve somewhat different purposes. It may, however, be fruitful to apply ideas from one approach to the further development of the other one. The papers of Lehmann and Smyth (LS 77) and Kanda (KA 78) represent different attempts to do this.

Recently, an algebraic analogon to type constructors, called parameterized data types, has been investigated (BG 77, EH 79, EL 79a+b, BG 80, EKTWW 80a+b). So, in a sense, algebraic data types can be specified by expressions involving parameterized data types applied to actual parameters. The question naturally arises whether a meaningful and useful analogon to recursive domain equations can be established in the algebraic framework. This paper shows that the answer is positive.

Our approach is inspired by the categorical versions of recursive domain equations due to Wand (WA 75), Lehmann (LE 76), and Smyth and Plotkin (SP 77). Thus, fixpoints

of functors play an important role. However, we do not need ω-continuity, and we have a simple syntactic method for solving domain equations. The solutions are algebraic data types, i.e. many-sorted algebras with all their operations.

2. FUNDAMENTAL NOTIONS

A signature is a quadruple $\Sigma=\langle S,\Omega, \text{arity}, \text{sort}\rangle$, where S and Ω are sets of sorts and operators, respectively, and $\text{arity}:\Omega \longrightarrow S^*$, $\text{sort}:\Omega \longrightarrow S$ are mappings. We will write $\Sigma=\langle S,\Omega\rangle$ for short, assuming tacitly the existence of the arity and sort mappings. A signature morphism $f:\Sigma \longrightarrow \Sigma'$ is a pair of mappings $f=\langle f_S:S \longrightarrow S', f_\omega:\Omega \longrightarrow \Omega'\rangle$ such that $\text{arity}(\omega f_\omega)=\text{arity}(\omega)f_S$ and $\text{sort}(\omega f_\omega)=\text{sort}(\omega)f_S$. For convenience, we often omit the index, writing f for f_S or f_ω.

Algebras are interpretations of signatures: a Σ-algebra A is an S-indexed family of sets, $\{s_A\}$, the carrier of A, together with an Ω-indexed family of mappings, $\{\omega_A:\text{arity}(\omega)_A \longrightarrow \text{sort}(\omega)_A\}$, the operations of A (if $x=s_1s_2\ldots s_n\in S^*$, x_A denotes the cartesian product $s_{1,A}\times\ldots\times s_{n,A}$). A Σ-algebra morphism $\varphi:A \longrightarrow B$ is an S-indexed family of mappings $\varphi_s:s_A \longrightarrow s_B$ such that, for each operator $\omega\in\Omega$ with arity x and sort s, we have $\omega_A\varphi_s=\varphi_x\omega_B$. Here, $\varphi_x=\varphi_{s_1}\times\ldots\times\varphi_{s_n}$ if $x=s_1\ldots s_n$. The class of all Σ-algebras with all Σ-algebra morphisms forms a category Σ-alg. It is well known that Σ-alg has an initial algebra I_Σ, having a unique morphism to any other algebra in Σ-alg.

If $f:\Sigma \longrightarrow \Sigma'$ is a signature morphism, there is a corresponding forgetful functor $f\text{-alg}:\Sigma'\text{-alg} \longrightarrow \Sigma\text{-alg}$ sending each Σ'-algebra B to that Σ-algebra A such that $s_A=(sf)_B$ and $\omega_A=(\omega f)_B$.

Let $\Sigma=\langle S,\Omega\rangle$ be a signature . A Σ-equation is a triple $\langle X, \tau_1, \tau_2\rangle$ where X is an S-indexed family of sets (of variables), and τ_1,τ_2 are terms over X and Ω of the same sort, called the sort of the equation. A Σ-algebra A satisfies a Σ-equation $\langle X,\tau_1,\tau_2\rangle$ iff the formula $\forall X:\tau_1=\tau_2$ is true when interpreted in A in the obvious way.

A specification is a pair $D=\langle\Sigma, E\rangle$ where Σ is a signature and E is an S-sorted set of Σ-equations. If $\Sigma=\langle S,\Omega\rangle$, we sometimes write $D=\langle S,\Omega, E\rangle$ instead of $D=\langle\Sigma,E\rangle$. A Σ-algebra satisfies a specification $D=\langle\Sigma,E\rangle$ iff it satisfies each equation in E. Σ-algebras satisfying a specification D will be called D-algebras. The full subcategory of Σ-alg consisting of all D-algebras is denoted by D-alg. D-alg, too, has an initial algebra, denoted by I_D.

A specification morphism $f:D \longrightarrow D'$ is a signature morphism $f:\Sigma \longrightarrow \Sigma'$ such that the corresponding forgetful functor $f\text{-alg}:\Sigma'\text{-alg} \longrightarrow \Sigma\text{-alg}$ sends each D'-algebra to a D-algebra. Thus, a specification morphism defines a forgetful functor also denoted by f-alg, but with D'-alg and D-alg as domain and range, respectively, obtained by restricting f-alg to D'-alg. The class of all specifications together with all specification morphisms forms a category denoted by spec.

It is not difficult to show that spec has all colimits, i.e. spec is cocomplete. We will use especially pushouts and coequalizers. A pushout is a square like that in figure 2.1(a) such that, whenever there are morphisms g_1, g_2 such that $f_1g_1=f_2g_2$, there is exactly one h such that $f_3h=g_1$ and $f_4h=g_2$. Given f_1, f_2, there are spec morphisms f_3, f_4 such that they form a pushout. We say that

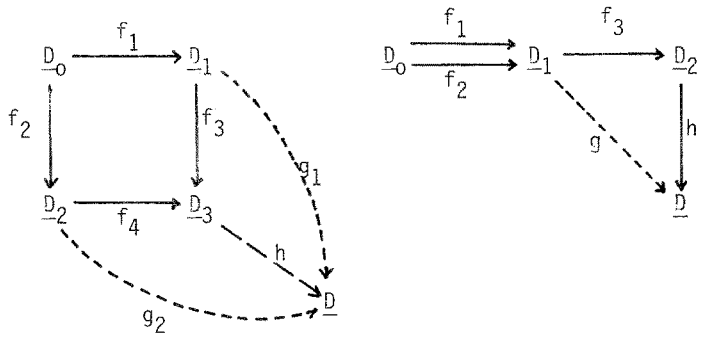

Figure 2.1

(f_3, f_4) is the pushout of f_1 and f_2. Analogously, a coequalizer is a diagram like that in figure 2.1(b) (without broken lines) such that, whenever there is a g such that $f_1g=f_2g$, then there is exactly one h such that $f_3h=g$. Given f_1 and f_2 as shown in figure 2.1(b), there is always a spec morphism f_3 forming a coequalizer. We say that f_3 is the coequalizer of f_1 and f_2.

Let $f:\underline{D} \longrightarrow \underline{D}'$ be a specification morphism. A functor $F:\underline{D}\text{-alg} \longrightarrow \underline{D}'\text{-alg}$ is called strongly persistent with respect to f iff $F.\underline{f}\text{-alg}$ is the identity on $\underline{D}$-alg. The following lemma is proven in EKTWW 80a+b:

Extension lemma: Let the pushout in figure 2.1(a) be given. Let $F_1:\underline{D_0}\text{-alg} \longrightarrow \underline{D_1}\text{-alg}$ be strongly persistent wrt f_1. Then there is exactly one functor $F_4:\underline{D_2}\text{-alg} \longrightarrow \underline{D_3}\text{-alg}$, called the extension of F_1 via f_2, that is strongly persistent wrt f_4 and satisfies $\underline{f_2}\text{-alg}.F_1 = F_4.\underline{f_3}\text{-alg}$.

Associated with a specification morphism $f:\underline{D} \longrightarrow \underline{D}'$, there is a functor f-free: $\underline{D}\text{-alg} \longrightarrow \underline{D}'\text{-alg}$ that is left adjoint to f-alg. f-free sends each $\underline{D}$-algebra A to the free $\underline{D}'$-algebra over A (wrt f). Also from EKTWW 80a+b, we have the following:

Extension lemma supplement: If, in addition, $F_1 \cong \underline{f_1}\text{-free}$, then we have $F_4 \cong \underline{f_4}\text{-free}$.

A parameterized specification is an injective spec morphism $p:\underline{X} \longrightarrow \underline{XP}$. X is the formal parameter of p. A parameterized data type is a pair (p,P) where $p:\underline{X} \longrightarrow \underline{XP}$ is a parameterized specification, and $P:\underline{X}\text{-alg} \longrightarrow \underline{XP}\text{-alg}$ is a functor. (p,P) is called strongly persistent iff P is strongly persistent wrt p. We call p strongly persistent iff $(p,\underline{p}\text{-free})$ has this property. In the case of strong persistency,

parameter passing works as follows. Given $p:\underline{X} \longrightarrow \underline{XP}$, an actual parameter for p is a pair $(f,\underline{D})$ where $\underline{D}$ is a specification and $f:\underline{X} \longrightarrow \underline{D}$ is a spec morphism. Let (p',f') be the pushout of p and f (cf. fig. 2.2). Let P' be the unique extension of P via f.

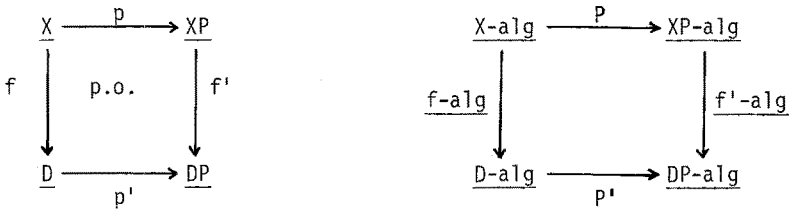

Figure 2.2

Then (p',P') is a strongly persistent parameterized data type, also called the extension of (p,P) via f. Now, each actual parameter $\underline{D}$-algebra A is sent to AP'. In this way, (p,P) works as a data type constructor, operating on various actual parameter algebras and preserving their structure.

Parameter passing works in the same way when the actual parameter is again parametric, i.e. $\underline{D}=\underline{YP}$ of a parametric specification $\hat{p}:\underline{Y} \longrightarrow \underline{YP}$. In this case, an (actual parameter) parametric data type $(\hat{p},\hat{P})$ is sent to the parametric data type $(\hat{p}p',\hat{P}P')$ where p',P' are obtained as above.

Example 2.1: The "n-product with constant"

$$\underline{X_1} \times \ldots \times \underline{X_n} + \underline{1}$$

is the embedding $p:\underline{X} \hookrightarrow \underline{XP}$, where $\underline{X}$ and $\underline{XP}$ are defined as follows $(i=1,\ldots n)$.

$\underline{X}$	rest of $\underline{XP}$
sorts $X_1,\ldots,X_n$,	P
ops $\bar{x}_i: \longrightarrow X_i$	$\bar{p}: \longrightarrow P$
$\equiv_i : X_i \times X_i \longrightarrow BOOL$	$\langle \_,\ldots,\_ \rangle : X_1 \times \ldots \times X_n \longrightarrow P$
	$\_(i) : P \longrightarrow X_i$
	$\equiv : P \times P \longrightarrow BOOL$
eqs $\bar{x}_i \equiv_i \bar{x}_i$ = true	$\bar{p}(i) = \bar{x}_i$
	$\langle x_1,\ldots,x_n \rangle (i) = x_i$
	$\bar{p} \equiv \bar{p}$ = true
	$\bar{p} \equiv \langle x_1,\ldots,x_n \rangle$ = false
	$\langle x_1,\ldots,x_n \rangle \equiv \bar{p}$ = false
	$\langle x_1,\ldots,x_n \rangle \equiv \langle x_1,\ldots,x_n \rangle = x_1 \equiv x_1 \ \ldots \ x_n \equiv x_n$

Consider (p,p-free) for n=2. Let D be a specification of the integers (sort INT) and booleans (sort BOOL). Let f:X $\longrightarrow$ D be defined by $X_1 \longmapsto$ BOOL, $X_2 \longmapsto$ INT, $\bar{x}_1 \longmapsto$ false, $\bar{x}_2 \longmapsto$ 0, $\equiv_i \longmapsto$ identity on BOOL or INT, respectively, i=1,2. Then, the initial boolean-integer algebra (D-algebra) is sent to the algebra of (boolean,integer)-pairs with one additional constant, all other operations of XP, and all boolean and integer operations on the components retained. The specification of this algebra is DP, the pushout object of p and f. It is obtained from XP by substituting BOOL for X_1, false for $\bar{x}_1$, INT for X_2, 0 for $\bar{x}_2$,and the respective identities for $\equiv_i$. Therefore, a suggestive notation for DP is BOOL×INT + 1.

3. ALGEBRAIC DOMAIN EQUATIONS

In the order theoretic approach to data types, parameterized data types can be viewed as functors, domain equations as endofunctors, and their solutions as fixpoints of functors (WA 75, LE 76, SP 77).

In the algebraic approach, parameterized data types are essentially functors, too, but most often they are not endofunctors. Typically, the signatures of actual parameter and resultant algebras are different. In order to get endofunctors, we define algebraic domain equations to consist of a parameterized data type and a functor in the reverse direction. We restrict ourselves to free and strongly persistent parameterizations of the form (p,p-free) and to algebraic reverse functors of the form e-alg for some spec morphism e.

Definition 3.1: An algebraic domain equation is a pair of spec morphisms (p,e), p,e:X $\longrightarrow$ XP, such that p is a strongly persistent parameterized specification.

Let P=p-free, $\bar{P}$=p-alg, and $\bar{E}$=e-alg. There are two endofunctors, namely $P\bar{E}$ on X-alg and $\bar{E}P$ on XP-alg. A fixpoint is an object that is sent to an isomorphic one. It is immediate to see that the fixpoints of $P\bar{E}$ and $\bar{E}P$ are very closely related: A is a fixpoint of $P\bar{E}$ iff AP is a fixpoint of $\bar{E}P$, and vice versa.

For the definition of what we mean by a solution of an algebraic domain equation, we make use of the following result. Let (q,Q) be the coequalizer in spec of p and e,

$$X \underset{e}{\overset{p}{\rightrightarrows}} XP \xrightarrow{q} Q \quad ,$$

and let $\bar{Q}$=q-alg.

Theorem 3.2: If B is a fixpoint of $\bar{E}P$, then there is a unique (up to isomorphism) Q-algebra C such that B=C$\bar{Q}$.

It is convenient not to take fixpoints of $P\bar{E}$ or $\bar{E}P$ as solutions, but these uniquely associated Q-algebras.

Definition 3.3: A solution of an algebraic domain equation (p,e) is a Q-algebra C such that C$\bar{Q}$ is a fixpoint of $\bar{E}P$.

The main result can now be stated as follows.

Theorem 3.4: The initial $\underline{Q}$-algebra I_Q is a solution of (p,e).

The proof is rather involved and requires some more technical machinery to be developed. It will be published elsewhere. Clearly, I_Q is an initial solution, i.e. it is initial in the full subcategory of $\underline{Q}$-alg of all solutions of (p,e). A specification of I_Q is $\underline{Q}$, the coequalizer object of p and e. There is a simple construction for $\underline{Q}$, given p and e, based on the coequalizer construction in set applied to sorts and operators.

Example 3.5: Consider the n-product with constant from example 2.1 for n=1. Let

$$\underline{X} \underset{e}{\overset{p}{\rightrightarrows}} \underline{X} + 1$$

be defined by taking p as in example 2.1, and e sending sort X_1 to P, $\bar{x}_1$ to $\bar{p}$, and $\equiv_1$ to $\equiv$. Then the solution is I_Q where $\underline{Q}$ is the following specification obtained from $\underline{X} + \underline{1}$ by identifying X_1 and P, $\bar{x}_1$ and $\bar{p}$, and $\equiv_1$ and $\equiv$. For convenience, we rename P by N, $\bar{p}$ by 0, $\langle\_\rangle$ by succ, and $\_(1)$ by pred.

sorts N

ops
$$0: \longrightarrow N$$
$$succ: N \longrightarrow N$$
$$pred: N \longrightarrow N$$
$$\equiv : N \times N \longrightarrow BOOL$$

eqs
$$pred(0) = 0$$
$$pred(succ(n)) = n$$
$$0 \equiv 0 = true$$
$$0 \equiv succ(n) = false$$
$$succ(n) \equiv 0 = false$$
$$succ(n) \equiv succ(m) = n \equiv m$$

This is a specification of the natural numbers.

Example 3.6: Let the above specification be $\underline{N}$. Let $\underline{X} \times \underline{N} + \underline{1}$ be the parametric specification obtained from $\underline{X_1} \times \underline{X_2} + \underline{1}$ (example 2.1) by parameterized parameter passing with actual parameter $(f,\hat{p})$, where $\hat{p}: \underline{X} \hookrightarrow \underline{X} + \underline{N}$ and f sends X_1 to X (forgetting the index 1), X_2 to N, $\bar{x}_2$ to 0, and $\equiv_2$ to $\equiv$. Then, the algebraic domain equation

$$\underline{X} \underset{e}{\overset{p}{\rightrightarrows}} \underline{X} \times \underline{N} + \underline{1}$$

has stacks as solutions, specified by the following specification (with obvious renamings):

sorts S,N
ops empty: → S
push : S * N → S
pop : S → S
top : S → N
= : S×S → BOOL
... (ops from N)
eqs pop(empty) = empty
top(empty)= 0
pop(push(s,n))= s
top(push(s,n))= n
... (eqs from N and eqs for ≡)

In a similar way, we get trees with a natural number attached to each node as solutions of the domain equation $\underline{X} = \underline{X} * \underline{X} \times \underline{N} + \underline{1}$, where = denotes an appropriate pair (p,e) of morphisms, etc.

4. PARAMETERIZED ALGEBRAIC DOMAIN EQUATIONS

Our theory so far gives algebraic data types as solutions of algebraic domain equations. It is natural to ask whether we can get parameterized data types as solutions of parameterized algebraic domain equations by a similar method of implicit specification and syntactic solution. This works indeed, if we proceed as follows.

Definition 4.1: A parameterized algebraic domain equation is a triple (r;p,e) of spec morphisms,

$$\underline{Y} \xrightarrow{r} \underline{XY} \underset{e}{\overset{p}{\rightrightarrows}} \underline{XYP}$$

such that (1) (p,e) is an algebraic domain equation, (2) r is a parameterized specification, and (3) rp=re. (r;p,e) is called strongly persistent iff r has this property.

Let (f,D) be an actual parameter for r. Then, using the mechanism of parameter passing as defined in the last section, we can construct an algebraic domain equation (p',e'), the (f,D)-instance of (r;p,e), as follows (cf. figure 4.1):

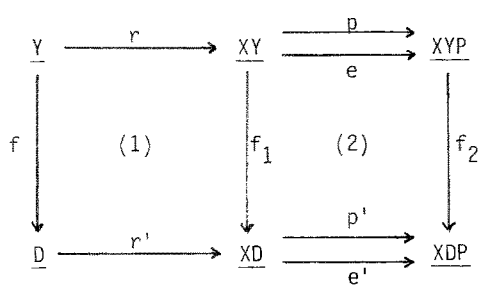

Figure 4.1

1) Let r', $\underline{XD}$,f_1 be such that (1) is a pushout.
2) Let p',$\underline{XDP}$,f_2 be such that (2) is a pushout wrt p,p'.
3) Define e' by r'e' = r'p' and $f_1e' = ef_2$. (e' is well defined since r' and f_1 are jointly surjective, and we easily prove that $rf_1e' = fr'e'$.)

Definition 4.2: A solution of a parameterized algebraic domain equation (r;p,e) is a parameterized data type (s,S) such that, for each actual parameter (f,$\underline{D}$) of r, (s,S) sends I_D via f to the initial solution of the (f,$\underline{D}$)-instance (p',e') of (r;p,e).

Of course, r and s must have the same source $\underline{Y}$. Our main theorem 3.4 now extends to the parametric case as follows.

Theorem 4.3: Let (r;p,e) be a strongly persistent parameterized algebraic domain equation, and let (q,$\underline{YQ}$) be the coequalizer of p and e. Then, if s=rpq, (s,$\underline{s\text{-free}}$) is a solution of (r;p,e).

Again, the proof is a little bit lengthy and will be published elsewhere. Examples of implicit parameteric specifications are $\underline{X}=\underline{X}\times\underline{Y} + \underline{1}$ (stacks over Y), $\underline{X}=\underline{X}\times\underline{X}\times\underline{Y} + \underline{1}$ (trees over Y), etc. Here, = denotes a pair (p,e) of morphisms that are defined like those in example 3.6.

5. CONCLUSIONS

The theoretical results presented here provide a sound and consistent semantics for a new algebraic specification method using parameterized specifications and algebraic domain equations. The feasibility and usefulness of this method for the development of specification methods and specification languages should be subject to further study.

Another possible area of application is the algebraic semantics of programming languages. In denotional semantics, domain equations are used extensively to specify the syntactic and semantic domains. Our theory can provide algebraic interpretations for them. There is, however, one difficulty: we get only "finitary" solutions, for example (initial) algebras of finite sets or finite functions. The central semantic domains of environments and states usually are finitary, so there seems to be no problem. It is, however, not quite clear how to cope with cases like procedure parameters. In particular, we cannot obtain a model for λ-calculus with our method, like Scott's reflexive domain (SC 72b). For these and similar cases, an extension of our theory to continuous algebras is necessary. This is subject to further study.

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