Exercise Sheet 01

(First Order Logic)

Please note: The exercises will be neither collected, corrected, nor graded.

Exercise 1 – Basics

All the sub-exercises should be answered with First Order Logic in mind.

a) Describe the relation between a language, an interpretation, and a system.
b) What is the difference between functions and predicates? Could you use functions instead of predicates or vice-versa?
c) What is the difference between a **term** and an **atom**?
d) What is an **open** and a **closed** formula?
e) What is a **rectified formula** and which problem does it address?
f) What is the difference between an interpretation and a substitution? Would it be a good idea to merge the **substitution** into the **interpretation**?
g) What is a closed world assumption and why is it often used in deductive databases?

Exercise 2 – Language Design

All the sub-exercises should be answered with First Order Logic in mind.

a) Design a first order **language** for simple arithmetic on natural numbers. One should be able to add numbers, subtract numbers, multiply two numbers, decide if a number is equal to another number, and if a number is greater than another number.
b) Provide an **interpretation** for your language of the previous sub-exercise.
c) Provide a formula for following statements:
    i) "5 is greater than 2"
    ii) "If x is greater than 0, then also x*y is greater than 0"
    iii) "x is either greater than y, or x is equal to y, or x is smaller than y"
    iv) "The sum of any two numbers is always smaller than the product of the same two numbers"
d) Evaluate the previous terms i-iv. Are they always true? Can they be true? If a term is not always true but can be true, provide an example substitution which makes it true.

Exercise 3 – Interpretation

All the sub-exercises should be answered with First Order Logic in mind.

a) Given is a language $\mathcal{L} = (\Gamma, \Omega, \Pi, \mathrm{X})$ with $\Gamma := \{a, b\}, \Omega := \{f(x), g(x, y)\}, \Pi := \{P, Q(x, y), R(x)\}$, and $\mathrm{X} := \{\mathrm{x}, \mathrm{y}\}$.
   i) Provide at least 10 (different) terms for $\mathcal{L}$.
   ii) Provide at least 6 (different) atoms for $\mathcal{L}$.

b) Are the following "strings" valid formulas with respect to $\mathcal{L}$?
   i) $f(g(x, y))$
   ii) P
   iii) $Q(x, y) \vee Q(a, b)$
   iv) $Q(g(f(a), x), f(y))$
   v) $\forall a(R(a))$
   vi) $\exists x(f(x))$
   vii) $R(x) \rightarrow \neg R(x)$
   viii) $\neg R(\neg R(f(x)))$