Deductive Databases & Knowledge Based Systems

Sheet 1

Exercise 1
All the sub-exercises should be answered with First Order Logic in mind. Answer briefly!

1. Describe the relation between a language, an interpretation, and a system.
2. What is the difference between functions and predicates? Could you use functions instead of predicates or vice-versa?
3. What is the difference between a term and an atom
4. What is an open or closed formula?
5. What is a rectified formula and which problem does it address?
6. What is the difference between an interpretation and a substitution? Would it be a good idea to merge the substitution into the interpretation?
7. What is closed world assumption and why is it often used in deductive databases?

Exercise 2
All the sub-exercises should be answered with First Order Logic in mind.

1. Design a first order language for simple arithmetic’s on natural numbers. One should be able to add numbers, subtract numbers, multiply two numbers, decide if a number is equal another number, and if a number is greater than another number.
2. Provide an interpretation for your language of the previous sub-exercise.
3. Provide a formula for following statements:
   a. “5 is greater than 2”
   b. “If x is greater than 0, then also x*y is greater than 0”
   c. “x is either greater than y, or x is equal to y, or x is smaller than y”
   d. “The sum of any two numbers is always smaller than the product of the same two numbers”
4. Evaluate the previous terms a-d. Are they always true? Can they be true? If a term is not always true but can be true, provide an example substitution which makes it true.
Exercise 3

All the sub-exercises should be answered with First Order Logic in mind.

1. Given is a language \( \mathcal{L} = (\Gamma, \Omega, \Pi, \mathcal{X}) \) with \( \Gamma = \{a, b\}, \Omega = \{f(x), g(x, y)\}, \Pi := \{P, Q(x, y), R(x)\}, \) and \( \mathcal{X} := \{x, y\}. \)
   a. Provide at least 10 (different) terms for \( \mathcal{L}. \)
   b. Provide at least 6 (different) atoms for \( \mathcal{L}. \)

2. Are the following “strings” valid formulas with respect to \( \mathcal{L} \)?
   a. \( f(g(x, y)) \)
   b. \( P \)
   c. \( Q(x, y) \lor Q(a, b) \)
   d. \( Q(g(f(a), x), f(y)) \)
   e. \( \forall a(R(a)) \)
   f. \( \exists x(f(x)) \)
   g. \( R(x) \rightarrow \neg R(x) \)
   h. \( \neg R(\neg R(f(x))) \)