Deductive Databases & Knowledge Based Systems

Sheet 3

Exercise 1
Please answer briefly, no novel-writing!

1. Explain unsatisfiable, satisfiable, universal. (3 points)
2. What is a model? What is a tautology? (2 points)
3. What is a semantic conclusion? What is semantic equivalence? (2 points)
4. What is an axiom? (1 point)
5. What is a clause? What is a Horn clause? (2 points)
6. What is special about the Herbrand interpretation? (3 points)

Exercise 2
1. Prove using the introduced Hilbert-style proof system following statement. (7 points)
   a. \( \vdash A \rightarrow A \)
   b. \( \vdash (A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C)) \)
   c. \( \vdash B \rightarrow ((B \rightarrow A) \rightarrow A) \)
2. Can the following formulas be written clauses? If so, provide the corresponding clause. Is it also a Horn clause? (5 points)
   a. \( A \rightarrow ((B \land C) \rightarrow D) \)
   b. \( (A \lor B \lor C) \rightarrow D \)
   c. \( (\neg A) \rightarrow (\neg B) \)
   d. \( (\neg A) \rightarrow C \)
   e. \( B \land (C \lor D) \)
3. Which of the following Hilbert interpretation for the language 
\[ L := \{\{a, b, c\}, \{f, g\}, \{p, q\}, \{ \} \} \] is also a \textbf{Herbrand} model the formulas \[ W := \{(p(a) \rightarrow p(b) \land p(b) \rightarrow p(c)) \rightarrow (p(a) \rightarrow p(c)), (p(a) \rightarrow p(b)) \rightarrow (p(b) \rightarrow p(a))\}\]? (3 points)
   a. \( I := \{p(a) \rightarrow p(b), p(b) \rightarrow p(c), p(a) \rightarrow p(c)\} \)
   b. \( I := \{p(a) \rightarrow p(b), p(b) \rightarrow p(a)\} \)
   c. \( I := \{p(a) \rightarrow p(b), p(a) \rightarrow p(c)\} \)