Next Lecture

7.1 Implementation of Datalog
7.2 Efficient computation of fixpoints

Knowledge-Based Systems and Deductive Databases
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Semantics of Datalog

• The Datalog semantics are given by 
  Herbrand interpretations
  – A Datalog program $\mathcal{P}$ is a set of Horn clauses
  – Any Herbrand interpretation that satisfies $\mathcal{P}$ is a model
  – Unfortunately, it is not quite that easy to compute an 
    Herbrand model for $\mathcal{P}$
  – Also, multiple models exist per program — which 
    conveys the intended semantic?

Semantics of Datalog

• Datalog$^f$ 
  – Datalog$^f$ is computationally complete
  – The intended semantic of a Datalog$^f$ program is 
    given by the least Herbrand model
    • For the least Herbrand model $\mathcal{M}$, $\mathcal{M} \subseteq \mathcal{M}'$ for any other 
      Herbrand model $\mathcal{M}'$ holds
    • This leads to $\mathcal{M} := \bigcap \mathcal{M}$, whereas $\mathcal{M}$ is the set of all 
      Herbrand models
    • Informally: The least model is a model for $\mathcal{P}$ and does not 
      contain superfluous statements

Semantics of Datalog

• Operational semantics for Datalog$^f$ 
  – To compute the least Herbrand model, a fixpoint 
    iteration approach can be employed
    • Start with an empty set of ground atoms
    • Iteratively refine set (by adding more atoms)
    • Fixpoint iteration is monotous (set is only expanded in 
      each iteration)
    • As soon as the fixpoint is reached, set becomes stable (i.e. 
      no changes)
    • The method is finite for Datalog$^f$
    • The stable result is equivalent to the least Herbrand 
      model

Semantics of Datalog

• Iterative Transformation step: 
  – Elementary production rule $T_\mathcal{P}$
    • Idea: Apply all given rules with premises contained in 
      the set of the previous step
    • For $I_0(\cdot)$, this puts all atoms into the result
    • For following steps, everything which can be followed by a 
      single application of any rule is added
Semantics of Datalog

• **Datalog**\textsuperscript{neg} is more difficult
  – Datalog\textsuperscript{neg} does not provide more expressiveness, but allows for more natural modeling
  – **Problems:**
    - Datalog\textsuperscript{neg} is potentially unsafe (i.e. generates infinite or excessively large models)
    - Datalog\textsuperscript{neg} is potentially ambiguous (i.e. multiple distinctive models possible)
      - No least Herbrand model possible
      - Instead, multiple minimal Herbrand Models with
      - Intersection of minimal models is not a model itself...

  **No least Herbrand model possible**
  – Instead, multiple minimal Herbrand Models with
  – Intersection of minimal models is not a model itself...

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Exercise 3

• **Stratify the following rules**
  – grandmother(X,Y) :- parent(X,Z), parent(Z,Y), female(Y).
  – mother(X,Y) :- parent(X,Y), female(Y).
  – father(X,Y) :- parent(X,Y), not(mother(X,Y))

  **Stratification**
  – $S_1 = \{\text{parent, female, grandmother, mother}\}$
  – $S_2 = \{\text{father}\}$

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Exercise 3

• **Given are facts of the following type**
  – baseproduct(name, price, weight)
  – product(name, componentname, amountOfComponents)

  baseproduct(rim,500,200).
  baseproduct(wire,10,5).
  baseproduct(rivet,5,1).
  ... product(climber,wheel,2).
  product(climber,lightframe,1).
  product(wheel,rim,1).
  product(wheel,spoke,32).

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Exercise 3

- Add some rules like these

\[ \text{prodetail}(X,Y,Z) :\] product\((X,Y,Z)\).
\[ \text{prodetail}(X,Y,Z) :\] prodetail\((X,Y_1,Z_1),\) prodetail\((Y_1,Y,Z_2),\) \(Z\) is \(Z_1*Z_2\).
\[ \text{prodpricedetail}(X,Y,Z) :\] baseproduct\((Y,Z_1,\_),\) prodetail\((X,Y,Z_2),\) \(Z\) is \(Z_1*Z_2\).

- Unfortunately, basic Datalog lacks aggregation facilities, thus price information is in its raw form…

Exercise 4

- Ackermann function

\[ A(n, m) = \begin{cases} n + 1 & \text{if } m = 0 \\ A(n - 1, A(n, m - 1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases} \]

- Datalog clauses

\[ A(n, m) = \begin{cases} n + 1 & \text{if } m = 0 \\ A(n - 1, A(n, m - 1)) & \text{if } m > 0 \text{ and } n > 0 \end{cases} \]

\[ \text{ackermann}(M,N,R) : M=0, R is N+1. \]

Exercise 4

- However, this Datalog program is potentially unsafe (more fun about that in this lecture)

7.1 Datalog\(\neg\)eg in Algebra

- In the previous week, we have seen the elementary production operator \( T_P \)
  - But how can we put this operator to use?
  - Many deductive DBMS choose to implement everything “from the scratch”
    - Especially implementations in Prolog and Lisp are very common
  - However, for reliably storing huge amounts of data (e.g. the facts in the extensional DB), there is already a wonderful technology: Relational Databases
    - Also, most applications already use RDBs and SQL
7.1 Datalog\textsuperscript{neg} in Algebra

- In this section, we will map Datalog\textsuperscript{neg} to Relational Algebra
  - This will allow us an implementation of Datalog concepts within a RDB
  - Also, this will allow us to take advantage of established features of databases
    - Query optimization
    - ACID properties
    - Load balancing
    - etc...

7.1 Relational Algebra

- When using the Relational Model and Relational Algebra, we assume the following:
  - Data (i.e. facts) is stored in multiple relations
  - A relation $R$ over some sets $D_1, \ldots, D_n$ is a subset of their Cartesian product
    - $R \subseteq D_1 \times \ldots \times D_n$
    - The sets $D_1, \ldots, D_n$ are finite and are called domains

7.1 Datalog\textsuperscript{neg} in Algebra

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7.1 Relational Algebra

- On those relations algebra operations are defined
  - Base operations of relational algebra
    - $\times$ Cartesian Product
    - $\sigma$ Selection
    - $\pi$ Projection
    - $U$ Set Union
    - $\setminus$ Set Minus
  - Derived operations
    - $\bowtie$ Joins ($R \bowtie S \equiv \sigma_\theta (R \times S)$)
    - $\bowtie_\left, \bowtie_\right$ Left & Right Semi Joins
      - $R \bowtie_\left S \equiv \pi_{\text{att}}(R) (R \bowtie S)$

7.1 Implementation

- In the following, we will implement the simple fixpoint iteration with relational algebra
  - We will only consider safe Datalog\textsuperscript{neg} programs, i.e. negative literals and head variables are positively grounded
  - Given are a safe Datalog\textsuperscript{neg} program $\mathcal{P}$ and a relational database
    - Each predicate symbol $r_1, \ldots, r_m$ symbol of the extensional database is assigned to a relation $R_1, \ldots, R_m$
      - e.g. those predicates providing the facts

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7.1 Implementation

- Each predicate symbol \( q_1, \ldots, q_m \) symbol of the intensional database is assigned to a relation \( Q_1, \ldots, Q_m \)
  - e.g. those predicates defined by rules
  - For ease of use, we restrict each predicate to either be defined in the intensional or the extensional DB
    - i.e. each predicate which was used to define facts is not allowed to occur in the head of a rule
    - This does not limit the expressiveness of Datalog programs

Just a short consideration:
We also sort positive literals before negative ones
This does not limit the expressiveness of Datalog programs

Transform all rules of the intensional DB such that the head contains only variables
This can be archived by replacing any head constant with a new variable and adding a literal binding that variable to the old value
- e.g. \( q(X, a, b) : \sim L_1, \ldots, L_n \)
  \[ \mathcal{H} := \{ \mathcal{L} \mid \mathcal{L} \in \{ \text{LT, GT, LTE, GTE, EQ, NEQ} \} \} \]
- Those relations are of infinite size and thus, of course, not stored the RDB
- We will see later that they can be removed

• Change the order of the variables such that their safety is ensured by the previous body literals
  - A literal is unsafe if it is potentially infinite
  - e.g. \( R \equiv X = Y, p(X), q(Y) \) is not in correct order as the safety \( X = Y \) is not ensured by previous literals
    - There are infinite possibilities for \( X \) being equal to \( Y \)
    - \( R \equiv p(X), q(Y), X = Y \)
      - is in correct order as \( p(X) \) and \( q(Y) \) limit the possible values of \( X \) and \( Y \)
    - We also sort positive literals before negative ones

• Each rule \( R : L_1, \ldots, L_n \) is now transformed to relational algebra as follows
  - For each literal \( L_i \), the respective atomic component \( A_i \equiv p_1(t_1, \ldots, t_m) \) is transformed into an relational expression \( B_i \)
    - \( B_i \equiv q(P) \) with \( P \) being the relation corresponding to \( p_1 \)
    - The selection criterion \( \theta \) is a conjunction of conditions defined as follows:
      For each \( t_i \) a condition is added
      - \( \#j = \#k \) if \( t_i \) is a constant symbol
      - \( \#j \neq \#k \) if \( t_i \) and \( t_k \) are the same variables

How could we map relational algebra to Datalog?
- \( \sigma_{a_2 \leq a_1} R \Rightarrow R(X, S) \)
- \( \pi_{X,Y} R \Rightarrow R'(X) : R(X, Y) \)
- \( R \times S \Rightarrow R(W, Y) : S(Y, Z), W=Z \)
- \( R \leftarrow [i \leq j, i \leq k] S \Rightarrow R(W, X) : S(Y, Z), W=Z \)
- \( R \leftarrow [i \leq j, i \leq k] S \Rightarrow R(W, X) : S(Y, Z), W=Z \)
- \( R \cup S \Rightarrow R'(X, Y) : R(X, Y), R'(X, Y) : S(X, Y), R'(X, Y) : \neg S(X, Y) \)
- \( R \setminus S \Rightarrow R'(X, Y) : R(X, Y), R'(X, Y) : S(X, Y), R'(X, Y) : \neg S(X, Y) \)
### 7.1 Implementation

- **Example:**
  
  \[ p(X, Z) : q(X, Y, Z), r(X, 1) \Rightarrow p(X, Z) : q(X, Z, Y), r(X, 1). Z = 2 \]

  \[ E_1 := \sigma_{(Z=2)}(Q) \]
  \[ E_2 := \sigma_{(Z=2)}(R) \]

  \[ E_3 := \sigma_{(Z=2)}(EQ) \]

- After treating the single literals, we will compose the body expression \( F \) from left to right:

  - Initialize the temporary expression \( F_1 := E_1 \)

- **Example:**

  \[ F_1 := F_1 \bowtie E_2 \text{ if } L_i \text{ is positive and shares variables with previous body literals} \]

  - \( \bowtie \) forces the columns representing the shared variables to be equal

  \[ F_1 := F_1 \bowtie Q \]
  \[ F_2 := \sigma_{(Q)}(R); F_3 := \sigma_{(Q)}(EQ) \]
  \[ F_3 := \sigma_{(Q)}(R) \times R; F_4 := \sigma_{(Q)}(EQ) \times R \]

  \[ F_2 := \sigma_{(Q)}(R) \times R; F_3 := \sigma_{(Q)}(EQ) \times R \]

  - In short: Conjunctions of unrelated literals result to computing the Cartesian Product

- **Example:**

  \[ F_1 := F_1 \setminus (F_1 \bowtie E_2) \text{ if } L_i \text{ is negative and shares variables with previous body literals} \]

  - \( \setminus \) forces the columns representing the shared variables to be equal

  \[ R := q(X, Y), X \leq Y \Rightarrow E_1 := F_1 \]
  \[ E_2 := \sigma_{(X \leq Y)}(Q) \]
  \[ E_3 := \sigma_{(X \leq Y)}(R); E_4 := \sigma_{(X \leq Y)}(EQ) \]
  \[ E_4 := \sigma_{(X \leq Y)}(EQ) \times R \]

  - In short: Conjunctions of related negative literals result in generating a join, using the related variables as join condition

- **Example:**

  \[ \text{Ex}((E_{prev} \bowtie EQ) \Rightarrow \pi_{\text{attributesOf}(EQ)}(E)) \]

  - This expression was created by e.g.: \( E(\ldots), X=c \)

- **Examples:**

  \[ R := q(X, Y), X \leq Y \Rightarrow \]

  \[ F := (\sigma_{(X \leq Y)}(Q) \times R) \times (Q \times R); \}

  \[ F := \sigma_{(X \leq Y)}(Q) \times R \]

  - By algebraic optimization, this will later result to

  \[ F := (\sigma_{(X \leq Y)}(Q) \times R) \]

  - This expression was created by e.g.: \( R(\ldots), X=Y \)
7.1 Implementation

- Finally, the whole rule \( C \equiv R \rightarrow L_1, \ldots, L_n \) is now transformed to the expression \( \text{eval}(C) := \pi_{\text{head}(t)}(F) \)
  - i.e. to evaluate the rule \( C \), we project all variables appearing in its head from its body expression \( F \)
- For evaluating one iteration step for given intensional predicate \( q_i \), all related rules have to be united
  - \( \text{eval}(q_i) := \bigcup_{C \in \text{def}(q_i)} \text{eval}(C) \)

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7.1 Implementation

- Now, the elementary production rule \( T_P \) corresponds to evaluating \( \text{eval}(q_i) \)
- Queries \( Q \equiv p(t_1, \ldots, t_n) \) can be transformed to relational algebra likewise
- Also note that Datalog can be translated to RelAlg + while Datalog \( \text{neg} \) has to be translated to full RelAlg
  
  – Negation requires the highly inefficient \( \setminus \) operator

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7.1 Implementation

- For actually performing the fixpoint iteration, following is performed
  1. Create tables for each intensional predicate \( q_i \)
  2. Execute the elementary production \( T_P \) (i.e. run \( \text{eval}(q_i) \) for each intensional predicate) and store results temporary
    a. If result tables are of the same size than the predicate tables, the fixpoint has reached and we can continue with step 3
    b. Replace content of intensional predicate tables with respective temporary tables
    c. Continue with step 2
  3. Run the actual query on the tables to obtain final result

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7.1 Implementation

- Example
  - \( \text{edge}(1, 2), \text{edge}(1, 3), \text{edge}(2, 4), \text{edge}(3, 4), \text{edge}(4, 5) \)
  - \( \text{path}(X, Y) := \text{edge}(X, Y) \)
  - \( \text{path}(X, Y) := \text{edge}(X, Z), \text{path}(Z, Y) \)
  - \( \text{path}(2, X) \)
    - The facts all go into the extensional table \( \text{Edge} \), an intensional table \( \text{Path} \) is created

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7.1 Implementation

- Execute elementary production on current tables
  - \( \text{eval}(\text{path}) := \text{Edge} \cup \text{Edge} \cup \text{Path} \)

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• Replace path table and repeat
  \[ \text{eval}(\text{path}) := \text{Edge} \cup \text{Edge} \times \left[ \text{left}.#2 = \text{right}.#1 \right] \]

\[ \text{edge} \]
\[ \begin{array}{cc}
1 & 2 \\
1 & 3 \\
3 & 4 \\
4 & 5 \\
\end{array} \]

\[ \text{path} \]
\[ \begin{array}{cc}
1 & 2 \\
1 & 3 \\
2 & 4 \\
3 & 4 \\
4 & 5 \\
\end{array} \]

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\end{array} \]

\[ \text{path} \]
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1 & 2 \\
1 & 3 \\
2 & 4 \\
3 & 4 \\
4 & 5 \\
\end{array} \]

7.2 Strategies

• Given an extensional database and a query, there are two general strategies for evaluation
  - Bottom-Up: Start with given facts in the EDB and generate all facts and discard those which don’t match the query
    - e.g. fixpoint iteration
    - Performs well in restricted and smaller scenarios
    - “forward-chaining”
  - Top-Down: Start with query and generate proofs down to the EDB facts
    - Most logical programming environments choose this approach
      - e.g. SDL resolution
    - Performs well in more complex scenarios in which bottom-up becomes prohibitive
      - “backward-chaining”
7.2 Strategies

- **Scenario**
  - All facts are within an extensional database EDB
  - All rules are within the Datalog program \( \mathcal{P} \)
    - No facts in \( \mathcal{P} \)
  - Given is a goal query \( Q \equiv p(t_1, \ldots, t_n) \)?

- **Bottom-up problems**
  - Generate all deducible facts of \( \mathcal{P} \cup EDB \)
  - When finished, throw away all facts not matching the query pattern.
    - All those facts whose predicate is not \( p \)
    - All those facts whose predicate is \( p \), but are more general than the query

- **Example with constants**:
  - \( Q \equiv p(a, X, b) \)?
  - Why should we generate all facts of \( p \) and later discard those which are not subsumed by \( Q \)?

- In the next lecture, we will explore bottom-up approaches which avoid generating unnecessary facts
  - Magic Sets
  - Count techniques

This lecture, we start with a simple top-down approach

7.2 Strategies

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7.2 Top-Down Evaluation

- **Basic Idea**:
  - Start with the query \( Q \equiv p(t_1, \ldots, t_n) \)?
  - Generate iteratively all proof trees ending with a ground instance of \( Q \) and starting with a known facts
    - Iterate over tree depth
    - As a helper data structure create all possible search trees of current depth
    - Transform search trees to all possible proof trees
    - Stop if no additional search trees / proof trees can be constructed

7.2 Top-Down Evaluation

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7.2 Top-Down Evaluation

- **Example**:
  - \( e(1, 2). e(1, 3). e(2, 4). e(3, 4). e(4, 5). e(5, 6). e(5, 7) \)
  - \( p(X, Y) \leftarrow e(X, Y) \), \( p(X, Y) \leftarrow e(X, Z), p(Z, Y) \) \( \) Rule 1
  - \( Q \equiv p(2, X) \), \( Q \equiv p(3, X) \), \( Q \equiv p(4, X) \), \( Q \equiv p(5, X) \), \( Q \equiv p(6, X) \), \( Q \equiv p(7, X) \) \( \) Rule 2

7.1 Top-Down Evaluation

- **Proof Trees of depth 0**
  - Which facts are ground instances of \( Q \)?
  - In our example, this is not the case for any fact...

- **Search Trees of depth 1**
  - Find all rules \( R \equiv B : A_1, \ldots, A_n \) such that \( Q \) and \( B \) are unifiable
    - Unifiable: There are substitutions such that \( B \) matches \( Q \)
  - For each rule \( R \), construct a search tree with \( Q \) as root
    - Attach a rule node to the \( Q \) containing \( R \)
    - Attach \( k \) subgoal nodes representing \( A_1, \ldots, A_n \) in its unified form
7.2 Top-Down Evaluation

- Search Trees of depth 1
  - Rule 1: $p(X, Y) \models e(X, Y)$.
  - Rule 2: $p(X, Y) \models e(X, Z), p(Z, Y)$.

To generate proof trees from a given search tree, we have to find a substitution $\rho$ such that for each goal node with clause $C, \rho(C) \in P \cup EDB$

- By applying this substitution to the whole tree, we obtain a proof tree

- The root node is a result of the query

Example:

- Find a substitution for $T_1$ ($T_2$ does not have one)

Example:

- For any $n>1$, all existing search trees of depth $n-1$ are expanded by treating any subgoal node as a goal node
  - Thus, new rule nodes and subgoals are appended

Example: Expanding $T_2$ to $T_{2,2}$ and $T_{2,1}$

- $T_{2,2}$

Example: $T_{2,1}$ and an substitutions $\rho$

Please note:

- By applying this type of backward-chaining, not all possible proof trees for the query can be generated
  - Only proof trees having ground facts in all leaf nodes are possible

• Those trees are called full proof trees
  - However, for each proof tree matching to the query, there is also an according full proof tree
We can see that the backward chaining proof trees can reach arbitrary depth.

- The backward chaining method is sound and complete.
- But consider the iterated use of rule 2.
  
  \[ \text{The tree is of infinite depth} \]

7.2 Top-Down Evaluation

- We can see that the backward chaining proof trees can reach arbitrary depth.
  
  - The backward chaining method is sound and complete.
  - But consider the iterated use of rule 2.

  \[ \begin{array}{c}
  q = p(2, X) \\
  p(Y, X) : - e(Y, Z), p(Z, X).
  \end{array} \]

  \[ \begin{array}{c}
  p(Z, X) : - e(Z, W), p(W, X).
  \end{array} \]

  \[ \begin{array}{c}
  p(W, X) : - e(W, V), p(V, X).
  \end{array} \]

  \[ \begin{array}{c}
  p(V, X) \end{array} \]

  \[ \vdots \]

  - The tree is of infinite depth.

When do we stop building trees?

- We have no idea a-priori which recursion depth we will need.
  
  \[ ? \text{path}(a, X) \]

  - Obviously, the more nodes we have, the deeper the recursion depth will be.

  - Still the number of sensible combinations of EDB facts and predicates in \( \mathcal{P} \) is limited since:
    
    - Both the database and the datalog program are finite.
    - We can only substitute any constant symbol from some fact in any predicate symbol at any position of a variable.

Proof sketch:

- \#predicates * \#constants^{max(args)} is an upper limit for the number of distinct ground facts derived from \( \mathcal{P} \) and EDB (purely syntactical).

- We can limit the production process to full proof trees, where at least one new fact is added in each depth level (otherwise the new level is useless...).

- Since we only have a limited number of ground facts, also the number of levels has to be limited...

Consider an example: a finite number of facts \{path(a,b), path(b,c),..., path(m,n)\} and a rule path(X,Y) :- path(X,Z), path(Z,Y).

- Worst case

  \[ \begin{array}{c}
  \text{Longest possible deduction chain is } \text{path}(a, n) \text{ of length } n-1 \\
  \text{The least determined query is } ?\text{path}(X, Y), \text{i.e. all paths} \\
  \text{There are } n \text{ constant symbols and a single predicate symbol} \\
  \text{The constants can occur in two places, i.e. max(args) = 2} \\
  \text{That means the maximum number of deducible facts is } n^2 
  \end{array} \]

Many backward-chaining algorithms rely on the concepts of search trees and proof trees.

- However, the generation strategy may differ.
  
  - In the previous example, the search trees have been generated one by one according to their depth.
  
  - This is called level saturation strategy and resembles an breadth-first approach.

- Alternatively, depth-first approaches are possible.
  
  - Rule saturation strategy.
7.2 Resolution

• The previously presented top-down algorithm is extremely naïve
  – It generates all possible search and proof trees up to the worst-case depth which are somehow related to the query
    • Performance is far from optimal
  – In case of less restricted scenarios (e.g. not only Horn clauses or infinite universes), this approach is inevitably doomed to failure

7.2 Resolution

• From the field of full logics, we can borrow the concept of resolution
  – A technique for refutation theorem proofing
    • “Reductio ad absurdum”
  – Mainly explored around 1965 by J.A. Robinson
  – Established itself as THE standard technique for logical symbolic computation

7.3. Recursive SQL

• There are several variants of resolution
  – Best known in the field of logical programming is the class of SDL resolution algorithms
    • “Linear Resolution with Selection Function for Definite Clauses”
    • Most popular among these are those general algorithms employed in languages like Prolog or Lisp
    • However, in the next lecture we shall study a simplified SDL resolution algorithm suitable for Datalog
      – Be curious – that will be fun!

7.3. Recursive SQL

• The research and developments in the area of deductive databases successfully provided the ability to perform recursive queries
  – And with these, some limited reasoning capabilities
  • However, most applications have been tailored to work with traditional SQL based databases
    – When using SQL2 (SQL-92), recursive queries cannot be facilitated without external control and huge performance penalties
    – SQL2 is still the default for most today’s databases

7.3. Recursive SQL

• SQL3 (SQL-99) is a later SQL standard which mainly aims at widening the scope of SQL
  – Contains many features which extend beyond the scope of traditional RDBs
    • Binary Large Objects
    • Limited support for soft constraints
    • Updatable views
    • Active databases
    • Object orientation
    • UDF / UDT / UDM
    • References
    • Recursive Temporary Tables

7.3. Recursive SQL

• Recursive temporary tables adopt many concepts of deductive databases into the SQL world
  – All vendors developed proprietary implementations of the recursive tables
    • Nobody cared for the standard…
    • Notation may differ
  – In DB2 known as Common Table Expressions
Main idea:
- **Predicates** are represented by temporary tables
- Usually, definition of temporary table consists of two parts which are united via union operator
  - **Base case**: Represents the extensional part of the predicates (i.e., known facts which are read from the database)
  - **Recursive step**: The intentional part encoding the rules

**Example: Paths in a graph**
- Prepare the edges

**Datalog**

```
edge(1,2). edge(1,3). edge(2,4). edge(3,4).
edge(4,5). edge(5,6). edge(5,7)
```

**SQL**

```sql
CREATE TABLE edge (x int, y int);
INSERT INTO edge VALUES (1,2),(1,3),(2,4), (3,4), (4,5), (5,6), (5,7);
```

**Datalog & SQL**

```
path(X, Y) :
  - edge(X, Y).
path(X, Y) :
  - edge(X, Z), path(Z, Y)
```

- Create a non-recursive view
  - and query all paths from 5
    - … which is quite boring
  - **Datalog & SQL**
    - `WITH path(x, y) AS (SELECT x, y FROM edge)`
    - `SELECT x, y FROM path WHERE x=5`

- Create a recursive view
  - and query it
  - **Datalog & SQL**
    - `WITH path(x, y) AS (SELECT x, y FROM edge)
      SELECT x, y FROM path WHERE x=5`
    - **DB2 Syntax**
      - Base Case
        - Recursive Step
      - `WITH path(x, y) AS (SELECT x, y FROM edge UNION ALL
        SELECT x, y FROM path WHERE x=4)
        SELECT x, y FROM path WHERE x=4;`
7.3. Recursive SQL

• Linear & Non-Linear Recursion
  – The SQL3 standard only specifies linear recursion
    • i.e. a recursive step definition may refer to its own recursive table only once
    • e.g. WITH path (x, y) AS (... UNION ALL SELECT e.x, p.y FROM edge e, path p WHERE e.y = p.x)
  – However, a DB vendor may decide to additionally support non-linear recursion
    • Fixpoint during evaluation may be reached faster
    • Evaluation more complex in general
    • e.g. WITH path (x, y) AS (... UNION ALL SELECT p1.x, p2.y FROM path p1, path p2 WHERE p1.y = p2.x)

• Common table expressions also support negation
  – However, restrictions similar to Datalog apply
    • Statement must be stratified
    • Negative references to tables must be positively grounded

CREATE TABLE toll (x int, y int):
  INSERT INTO toll VALUES (1, 2);

WITH path (x, y) AS (SELECT x, y FROM edge UNION ALL SELECT e.x, p.y FROM edge e, path p WHERE e.y = p.x),
goodpath (x, y) AS (SELECT x, y FROM edge e WHERE NOT EXISTS (SELECT t.x, t.y FROM toll t WHERE t.x = e.x AND t.y = e.y) UNION ALL SELECT p1.x, p2.y FROM goodpath p1, goodpath p2 WHERE p1.y = p2.x)
SELECT 1, y FROM goodpath

Careful: This is not linear (e.g. won’t work in DB2)

Next lecture

• More implementation and optimization techniques
  – Magic Sets
  – SDL resolution
  – Further optimization