Exercise Sheet 01

Please note: The exercises will neither be collected, corrected, nor graded.

Exercise 1 – Basics (First Order Logic)

a) Describe the relation between a language, an interpretation, and a system.
b) What is the difference between functions and predicates? Could you use functions instead of predicates or vice-versa?
c) What is the difference between a **term** and an **atom**?
d) What is an **open** or **closed** formula?
e) What is a **rectified formula** and which problem does it address?
f) What is the difference between an interpretation and a substitution? Would it be a good idea to merge the **substitution** into the **interpretation**?
g) What is a closed world assumption and why is it often used in deductive databases?

Exercise 2 – General Questions (Logical Model)

a) Explain the terms: **unsatisfiable**, **satisfiable**, **universal**.
b) What is a **model**? What is a **tautology**?
c) What is a semantic **conclusion**? What is a semantic **equivalence**?
d) What is an **axiom**?
e) What is a **clause**? What is a Horn **clause**?
f) What is special about the **Herbrand interpretation**?

Exercise 3 – Datalog Basics

a) The following Boolean formula is given:

$$W \equiv (\neg x_1 \lor x_2 \lor \neg x_3) \land (x_2 \land x_4 \to x_5) \land x_4 \land \neg(x_5 \land x_6 \land x_7) \land (x_6 \lor \neg x_2) \land (x_1 \land x_3)$$

Is W satisfiable or not? If so, provide a substitution which satisfies it.
b) What does it mean if "A W formula is not decidable within a Theory (deductive system) T" ?
c) What are $Datalog^{f,neg}$, $Datalog^f$, $Datalog^{neg}$ and $Datalog$ in comparison?
d) What is "stratification"? Why is it needed? Which problem does it address?
e) Stratify the following rules:

grandmother(X,Y) :- parent(X,Z), parent(Z,Y), female(Y).
mother(X,Y) :- parent(X,Y), female(Y).
father(X,Y) :- parent(X,Y), not(mother(X,Y)).

Exercise 4 – Interpretation

All the sub-exercises should be answered with First Order Logic in mind.

a) Given is a language $\mathcal{L} = (\Gamma, \Omega, \Pi, X)$ with $\Gamma := \{a, b\}, \Omega := \{f(x), g(x, y)\}, \Pi :=$ $\{P, Q(x, y), R(x)\}$, and $X := \{x, y\}$.

    i)   Provide at least 10 (different) terms for $\mathcal{L}$.

    ii) Provide at least 6 (different) atoms for $\mathcal{L}$.

b) Are the following "strings" valid formulas with respect to $\mathcal{L}$?

    i) $f(g(x, y))$

    ii) P

    iii) $Q(x, y) \vee Q(a, b)$

    iv) $Q(g(f(a), x), f(y))$

    v) $\forall a(R(a))$

    vi) $\exists x(f(x))$

    vii) $R(x) \rightarrow \neg R(x)$

    viii) $\neg R(\neg R(f(x)))$